

Self-Dual Superstring in Six Dimensions¹

John H. Schwarz

*California Institute of Technology, Pasadena, CA 91125 USA***Abstract**

A free superstring with chiral $N = 2$ supersymmetry in six dimensions is proposed. It couples to a two-form gauge field with a self-dual field strength. Compactification to four dimensions on a two-torus gives a strongly coupled $N=4$ four-dimensional gauge theory with $SL(2, \mathbb{Z})$ duality and an infinite tower of dyons. Various authors have suggested that this string theory should be also the world-volume theory of M theory five-branes. Accepting this proposal, we find a puzzling factor of two in the application to black-hole entropy computations.

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Evidence has been accumulating for the existence of a new kind of superstring theory. This string theory would be non-gravitational, defined in a rigid six-dimensional background geometry. Also, it would be self-dual in the sense that it carries a charge that couples to a two-form B with a self-dual field strength H ($H = dB = *H$). Thus, encircling the string with a three-sphere S^3 , the magnetic charge $\int_{S^3} H$ and the electric charge $\int_{S^3} *H$ are one and the same.² This string is also supposed to give a 6D theory with chiral $N = 2$ supersymmetry. By analogy with the convention in ten dimensions, I propose to refer to this as a self-dual type IIB string theory.

The first evidence for existence of this theory was put forward by Witten [2]. He considered the ten-dimensional IIB superstring theory compactified on $\mathbb{R}^6 \times K3$, which gives a six-dimensional theory with IIB supersymmetry. Among the various BPS p -brane solitons, he drew special attention to those that arise from wrapping the self-dual three-brane on a two-cycle of the $K3$. This appears as a self-dual string in \mathbb{R}^6 with IIB supersymmetry and a tension that is proportional to the area of the two-cycle. By approaching a point in the moduli space that corresponds to this area vanishing, the string tension can be made arbitrarily small. Thus, the scale of the tension can be decoupled from the Planck scale, and there should be an effective description of the dynamics of this string in a rigid space-time background. The massless modes of the string are easily identified as a IIB tensor multiplet, which consists of a two-form with self-dual field strength, five scalars, and two chiral fermions. It is convenient to describe states by representations of the little group $SO(4) \approx SU(2) \times SU(2)$. In this notation, the tensor multiplet contains $(3, 1) + 5(1, 1) + 4(2, 1)$.

A second piece of evidence for the existence of this string theory comes from M theory. This (still mysterious) theory in eleven dimensions is known to contain a BPS two-brane and a dual five-brane. Moreover, it has been noted that a two-brane can end on a five-brane, where its boundary looks like a closed string [3]. This is analogous to the story for D -branes, where (by definition) open strings can end. Indeed, if one imagines an open string ending on a Dirichlet 4-brane in the IIA superstring theory, and recalls that this theory actually contains a circular eleventh dimension, it is clear that this configuration corresponds at strong coupling to a M theory two-brane ending on a five-brane.³ This picture has been utilized to argue that the massless modes of the five-brane world-volume theory should consist of

²This possibility was first noted in Ref. [1].

³It is less clear to me what eleven-dimensional interpretation should be assigned to a IIA open string ending on Dirichlet two-brane.

a IIB tensor multiplet [4]. Moreover, it has been suggested that the complete five-brane world-volume theory should be a self-dual superstring theory with exactly the properties discussed above [5].

Because the superstring charge is self-dual, the two-form field couples to the string with strength of order unity; therefore, the theory in question is necessarily strongly coupled. Without a weak coupling limit analogous to that of more familiar string theories, one might wonder whether it is possible to say anything sensible about this theory beyond its massless spectrum. The key to a large portion of recent progress has been to focus on BPS saturated states whose properties at weak coupling often extend unchanged to strong coupling. This type of reasoning will play a role in our reasoning later, when we consider compactifying some of the six dimensions, but in flat infinite six-dimensional space it is not so useful. Rather, I would like to suggest that even though extended classical strings interact strongly, the individual quantum states of the string do not carry the B charge, and it is consistent to treat them as free particles. One way of thinking about this is to imagine compactifying to four dimensions on $\mathbb{R}^4 \times T^2$. Then, as noted in Ref. [2], the B field gives rise to a $U(1)$ gauge field in four dimensions. Moreover, winding numbers around the two cycles of the torus correspond to electric and magnetic charge. An attractive aspect of this picture is that the $SL(2, \mathbb{Z})$ duality of the resulting $N=4$ four-dimensional gauge theory acquires a geometric interpretation in terms of the torus. This theory, containing both electrically and magnetically charged states, is certainly strongly coupled. However, in the decompactification limit all the charged states become infinitely massive. Therefore, it seems consistent to consider the \mathbb{R}^6 string as noninteracting.

With the motivation described above, let us now try to construct the desired free superstring theory. It is natural for this purpose to work in the space-time supersymmetric (“Green–Schwarz”) formalism. When that formalism was first introduced, it was noted [6] that at the classical level the construction work equally well for $D = 10, 6, 4, 3$. However, manifestly covariant quantization is extremely difficult, because of the structure of constraints. Quantization is straightforward in a light-cone gauge, but then it is necessary to check whether Lorentz invariance is preserved [7]. It was asserted that this is the case for $D = 10$ only. Now that a 6D theory is desired, one wonders what might have been overlooked.

If one does the usual light-cone gauge analysis in six dimensions, the left-moving or right-moving part of the string spectrum is generated by bosonic oscillators $\{\alpha_n^i\}$, which are the

modes of the transverse spatial dimensions, and fermionic oscillators $\{S_n^a\}$, which are modes of the surviving components of the θ coordinates. These have the usual algebras

$$\begin{aligned} [\alpha_m^i, \alpha_n^j] &= m\delta_{m+n,0}\delta^{ij} \\ \{S_m^a, S_n^b\} &= \delta_{m+n,0}\delta^{ab}. \end{aligned} \tag{1}$$

The index i labels the $(2, 2)$ representation of $SU(2) \times SU(2)$ and the label a labels the $2(2, 1)$ representation. Since there are four of each, the zero-point energies cancel, and the ground state is massless. The left-moving or right-moving component of the massless spectrum is given as a representation of the zero-mode Clifford algebra $\{S_0^a, S_0^b\} = \delta^{ab}$. This gives a multiplet $2(1, 1) + (2, 1)$, which is one-half of a $N = 1$ hypermultiplet. This is a representation of $N = 1$ supersymmetry, but it is not CPT invariant. This would be a problem if we were constructing an open string theory, but it need not be one for a closed-string theory. The massless IIB spectrum is given by tensoring this multiplet with itself, and this gives exactly the desired IIB tensor multiplet, which is CPT invariant.

One might wonder at this point whether this six-dimensional theory is consistent after all. To see that it is not, let us consider the first massive level. This is obtained by applying the raising operators α_{-1}^i and S_{-1}^a to the ground state and gives the $SU(2) \times SU(2)$ right-moving content

$$\left((2, 2) + 2(2, 1)\right) \times \left(2(1, 1) + (2, 1)\right). \tag{2}$$

Since this describes a massive level, these states should combine into $SO(5) \approx USp(4)$ multiplets. Such multiplets are non-chiral, which means that they are invariant under interchange of the two $SU(2)$'s. This clearly fails here, so we confirm the claim of Ref. [6] that this is not a Lorentz invariant theory.

The next step is to consider how the preceding construction could be modified so as to recover 6D Lorentz invariance without changing the massless sector. The key to answering this question is to recall that the corresponding light-cone gauge construction in ten dimensions succeeds. It contains bosonic coordinates transforming as an 8-vector of $SO(8)$ and fermionic coordinates transforming as an 8-spinor of $SO(8)$. With respect to the $SU(2) \times SU(2)$ subgroup considered here, the 8 bosons decompose as $(2, 2) + 4(1, 1)$ and the 8 fermions decompose as $2(2, 1) + 2(1, 2)$. Since we already have oscillators α_n^i and S_n^a corresponding to the $(2, 2)$ and $2(2, 1)$, this suggests introducing new ones corresponding to $4(1, 1)$ and $2(1, 2)$. The simplest way to avoid changing the zero modes is to take these fields to be

antiperiodic on the string. This means that the oscillators have half-integer modes. Thus, we introduce bosonic oscillators β_r^I and fermionic oscillators $T_r^{\dot{a}}$, where $r \in \mathbb{Z} + 1/2$, I labels $4(1, 1)$, and \dot{a} labels $2(1, 2)$. I now claim that the resulting spectrum has six-dimensional Lorentz invariance. Moreover, the chiral closed-string spectrum has IIB supersymmetry. One could check this by representing the super-Poincaré generators in terms of the oscillators and checking the algebra, as in [7]. This is somewhat tedious, and I have not bothered to do it. Instead one can note the following: if one were to compactify the ten-dimensional IIB superstring on the orbifold T^4/\mathbb{Z}_2 , the twisted sector associated with any one of the orbifold points would have exactly the content that we have described, and this is certainly part of a theory with 6D Lorentz invariance.⁴ Another easy check is to examine the first few massive levels, and to verify that they assemble into $USp(4)$ multiplets. For example, the first massive level has a right-moving spectrum given by acting with $\beta_{-1/2}^I$ and $T_{-1/2}^{\dot{a}}$ on the ground state. This has $SU(2) \times SU(2)$ content

$$(4(1, 1) + 2(1, 2)) \times (2(1, 1) + (2, 1)), \quad (3)$$

which assembles into two copies of the massive vector supermultiplet, whose $USp(4)$ content is $\mathbf{5} + 3 \cdot \mathbf{1} + 2 \cdot \mathbf{4}$.

We now seem to have a consistent free theory in six dimensions with the desired properties. As we have mentioned, toroidal compactification necessarily turns it into a strongly interacting theory. It would be very interesting to find an efficient way of describing string interactions in that case.

In Ref. [5] it was proposed that the M theory five-brane is described by a superstring theory of the type we have described. The significant difference is that Ref. [5] only included α_n^i and S_n^a excitations, and not β_r^I and $T_r^{\dot{a}}$ excitations. By compactifying M theory on T^5 or T^6 , they computed the BPS 0-brane spectrum in six and five dimensions. The five-brane wraps on the torus, and its modes are described as multi-string configurations. The BPS spectrum they found in six dimensions only depends on the self-dual string ground state; therefore, it is not affected by the addition of β and T modes. However, the five-dimensional spectrum utilizes the entire left-moving string spectrum. Specifically, Ref. [5] used the fact that the asymptotic density of states is characterized by a unitary conformal field theory with $c = 6$. This entered into the standard asymptotic degeneracy formula $\exp(2\pi\sqrt{ch/6})$, where h is the level number. The addition of β and T oscillators implies that one should

⁴This construction suggests possible generalizations.

take $c = 12$ instead. Using $c = 6$, Ref. [5] obtained a $D = 5$ black-hole entropy in agreement with that obtained previously from other considerations. This result has been confirmed recently for four-dimensional black holes, as well [8]. In view of the results presented here, it is puzzling why $c = 6$ rather than $c = 12$ should give the correct entropy.

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